The Castle of Groups Interview with Pierre Cartier*

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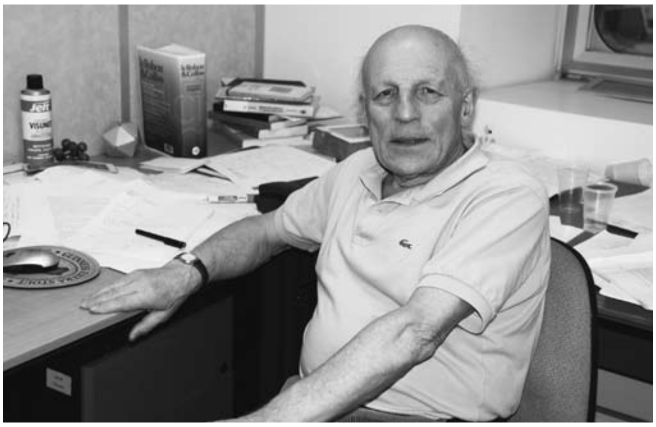

Pierre Cartier

"I have just met a wonderful person. He understands everything, has an insatiable curiosity and an outstanding openness of mind. Indeed, he has a sporty look and comes to Bures by bike." These are the words with which Alain Connes described to his wife his first impression of Pierre Cartier 30 years ago, the same as those he still gives today to the mathematicians who attend his lectures on topics as diverse as operad theory, algebraic groups, multizeta functions and functional integration. A former alumnus of the École Normale Supérieure, where he wrote his thesis with Henri Cartan, Pierre Cartier was deeply involved in Bourbaki's group from 1955 to 1980. In Algebraic Geometry, the duality between abelian varieties and formal groups, as well as the divisors defined by an open cover and a family of rational functions all carry his name.

After finishing his thesis, he stayed for two years at the Institute for Advanced Studies. He then taught at the University of Strasbourg before becoming a professor at the Institut des Hautes Études Scientifiques. A Renaissance man, Cartier is passionate about music, philosophy and literature, as is illustrated by his choice of the title *A Mad Day's Work* for an article exploring the evolution of the idea of space. At the beginning of this interview he goes back to his first *souvenirs d'apprentissage* and talks about Grothendieck, who was a very close friend of his for a long time. In the second part, we approach the not so easy relationship between physics and mathematics and the mysterious "cosmic Galois group" which he named.

School years

After your first education at Sedan and the Lycée Saint-Louis, you entered the École Normale in 1950, when Henri Cartan was a professor there. What are your memories of the discovery of real mathematics?

During my youth, Sedan was destroyed by the war, it was really hard to get food and the school survived in very precarious conditions. When I entered it in 1942, it operated more or less normally but there were very few books. It was a brother of my grandmother and a teacher in the elementary school who taught me the very beginning of algebra and elementary geometry. Very soon after, I was reading all the books I could lay hands on. I remember that I had great difficulty understanding a very bad textbook which explained relativity without any mathematics. It was a completely self-taught work. For instance, I skipped through the encyclopedias looking for articles about mathematics. I remember the entry "Abelian: There are three types of Abelian integrals: first kind, second kind, third kind". But what was an Abelian integral?

At 16, I won the first prize of the *concours général* and a friend of my parents decided to make me a present. He took me to a bookstore in the Latin Quarter, he whispered the amount he would pay for me to the owner and then he left. When he came back two hours later, I had chosen a book on tensor calculus by André Lichnerowicz and the first volume of Bourbaki's topology, which was hard for me to read because I was not used to the notation of set theory. When I arrived in Paris, the education we received at the Lycée Saint-Louis was quite solid but old-fashioned. However, I had good teachers and in my spare time I could read Chevalley's book on Lie groups and the works of Hermann Weyl in German. So when I entered the École Normale, I already had an idea of what the mathematics of the 20th century was about.

Entering Bourbaki

At the end of your first year at the École Normale you were invited to a Bourbaki meeting. What was the group like when you joined it for the first time?

During my first year at the École Normale, I attended all the courses I was able to. Samuel Eilenberg had been invited for the whole year by Henri Cartan, with whom

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Jean Leray (second from left), Jean Dieudonné (left) and Pierre Cartier (right), 1985. Photograph by Gerd Fischer. Archives of the Mathematisches Forschungsinstitut Oberwolfach.

he was writing *Homological Algebra*, so I learned a lot from him. But I also took physics and philosophy courses and seminars. At the end of the year I had to choose: Althusser, the Marxist, advised me that it was better to take the mathematics rather than the philosophy exams, Yves Rocard proposed to me that I help build the French atomic bomb and Henri Cartan invited me to a Bourbaki meeting.

It was the beginning of the best years of Bourbaki, when each book added something new and people waited for its release. The initial project ended in 1940 with the publication of the first volumes about Set Theory and Topology. Then the war came and many of the Bourbaki members had to escape from the Nazis. In spite of the very difficult circumstances, the group managed to continue working during those years. In 1950 a new generation, whose natural leader was Serre, had taken control. In that period we had huge ambitions; Bourbaki really wanted to write down all of mathematics. For me it was a dazzling experience; I learned so many things during the week I spent with them. According to Bourbaki's method, we studied reports on the topics which were to be treated in the series. Moreover, it was there where I met André Weil, to whom I kept very close during the rest of his life.

In fact, two of the founding fathers of Bourbaki, Henri Cartan and André Weil, later become your PhD advisors.

Officially my advisor was Roger Godement, who was at Nancy, but I was more inspired by the ideas of Cartan and Weil, so I decided to change subject. André Weil had a position in the United States but he came back every summer and I always took the opportunity to explain to him the advances on my thesis. In the winter of 1952, he had given a course on adeles and ideles. I took notes on the lectures and I helped to simplify some proofs and to add some supplementary notions. This allowed me to really get in touch with André Weil.

In addition, I had been reading his book *Foundations of Algebraic Geometry* word by word until the revolution of Serre, after which Weil told me: "I see that my *Foundations* are outdated". The best result of my thesis solved a problem posed by Weil in his book on Abelian varieties and algebraic groups. One day I had an inspiration: I was aware of what Dieudonné was doing with formal groups, I also had in mind the question posed by Weil and I said to myself: "This is linked". I saw it immediately but it took a very long time to prove because it lies on the crystalline cohomology of schemes (to be developed only after 1965).

I have never abandoned Group Theory; for me it is still the central point of everything on which I have worked. In my opinion, a compulsory reading was *Group Theory and Quantum Mechanics* by Hermann Weyl, a text which I still read today with the same interest. Group Theory is crucial in physics, geometry and arithmetic. For me it is like having a fortress: the castle of groups. I can go from here to there and maybe ring another doorbell but I always come back to my castle.

You were the secretary of Bourbaki from 1970 to 1983, after Dieudonné, Samuel and Dixmier. What was the working method?

The way of working was well-established: there were one week meetings in spring and in autumn and a two week meeting in the summer. For twenty-five years, I devoted a third of my mathematical career to Bourbaki. It is a lot! We published two new volumes every year and each book was rewritten several times. To start the process of writing, we asked the member who best mastered the topic to make a report on the most important theorems. From there, we planned what we wanted to explain and the relations to what we had already published and what we were going to present in the future. After that, somebody wrote a first draft, which we never liked.

Sometimes the process could take up to eight years, with plenty of changes, until the moment when Dieudonné knocked on the table: "It is finished, it has to be published". Then he took all the documents of the process and in the course of two months he did the synthesis, added all the exercises, sent the book to the printer and corrected the proofs. Dieudonné, with this amazing capacity for work, was the secret of Bourbaki. He said: "I do not work too much. I only write five pages a day between five and eight in the morning". But every day, every week, every year over sixty years, that makes a total of 110,000 pages. In fact, when he left Bourbaki, everything became harder.

The divorce between physics and mathematics

To people who talk about the destructive influence of Bourbaki's books, what would you say is the unquestionable heritage of the group?

The ambition of Bourbaki was to provide an encyclopedia which showed that there is only one mathematics and not several branches. That was the reason why we wanted to write *Élements de mathématique*, in the singular. According to Thomas Kuhn's theory on the structure of scientific revolutions, there always exist two periods: the scientific revolution itself, in which new questions are posed and new methods are invented, and the periods of normalization, when a paradigm is created which lasts until the next revolution. Bourbaki contributed to this stage of normalization, after the great conceptual revolution of mathematics in which Set Theory, Topology, Operator Theory and Modern Algebra were created.

However, when consolidating a theory, sometimes new ideas also appear. For instance, Bourbaki deeply changed Ricci's tensor calculus. In Commutative Algebra, in spite of the fact that 400 pages in the line of Zariski-Samuel were almost in print, Bourbaki restarted everything following the ideas of Serre and Grothendieck on localization, the spectrum of a ring, the filtration and topologies in Commutative Algebra and so on. It is also commonly accepted that the books on Lie theory anticipated the development of p-adic and algebraic groups, which is a landmark of success.

What about education?
I am much more critical in that respect. Bourbaki is an encyclopedia, not a textbook. In a religion the founder can be a great man but disciples do not always match up to the master. In the 70s, many extremist disciples of Bourbaki, who were not generally creative mathematicians, wanted to found an education scheme which began, starting even from kindergarten, from the most rigorous basis. The result was the five-year-old girl who went back home saying: "I do not want to be a set". Another consequence of the evident excess of abstraction in pedagogy is that nowadays the balance has moved to the other side and the idea of what a proof means is not considered important. In mathematics there are facts and proofs and both are equally worthy.

You tried to introduce mathematical physics, a subject in which you have been interested since you were young, to the Séminaire Bourbaki. How do you explain the refusal of many of the members?
It is really hard for me to understand André Weil's contempt for physics, since he was in Göttingen the year that Quantum Mechanics was born and he admired the work of Riemann, Gauss, Euler and Fermat, who were as much physicists as they were mathematicians. This may be due to the supremacy of experimental over theoretical physics in France, where Mechanics disappeared completely from universities with a reform around 1950. In the *Séminaire*, given my position in the group, I could choose any topic which I wanted, but every time that I lectured on physics I felt some kind of resistance.

In Grothendieck's case it was mostly an ideological reason. For him, who was extremely anarchistic, physics formed part of the military-industrial complex and there was an evident equation 'physics = atomic bomb'. In fact, when he left the IHÉS, in a very rude manner, it was under the pretext of military funding which his institute was receiving. It could have been very easy to keep the peace but Grothendieck did not want it. I remember that I found myself once between him and the police chief of Nize; he wanted to go to prison and the chief refused to arrest Grothendieck!

Grothendieck

What was the atmosphere at the IHÉS like during the years in which the* Eléments *and the* Séminaires de Géométrie Algébrique *were written?
In my opinion there are two miracles which explain Grothendieck's success in Algebraic Geometry. As David Mumford told me humorously, there was on the one hand the Zariski school in the United States, which had obtained many results using the method of resolution of singularities but which had already reached the limit. "We were a group which had problems without methods, and on the other side Grothendieck had methods without problems." So Zariski had the enormous generosity of sending all his students to learn Grothendieck's ideas and the IHÉS became an annex of Harvard and Princeton. The second miracle consisted of a completely improbable collaboration between three very different people: Grothendieck, a prophet who was more interested in general ideas than in the details, Serre, an extremely logical spirit, precise, no nonsense, and Dieudonné, with his extraordinary capacity for work and understanding. The *Éléments* and the *Séminaires de Géométrie Algébrique* are the result of this trio, on which no one would have bet.

What was the ontological status of categories for Grothendieck?
For him, categories are the language of mathematics and there is no other: we could not even state the problems solved by Grothendieck without talking about categories and functors. His aim was to prove the Weil conjectures by constructing a homology theory satisfying some categorical properties. Next he invented a concept of which he was prouder: topos, which he considered the last truth about space. In my opinion, the idea of topos is very important but it does not exhaust the conception of space. For instance, Connes' noncommutative geometry is another very good illumination.

Ontological, ontological? Nowadays, one of the most interesting points in mathematics is that, although all categorical reasonings are formally contradictory, we use them and we never make a mistake. Grothendieck provided a partial solution in terms of universes but a revolution of the foundations similar to what Cauchy and Weierstrass did for analysis is still to arrive. In this respect, he was pragmatic: categories are useful and they give results so we do not have to look at subtle set theoretic questions if there is no need. Is today the moment to think about these problems? Maybe… I have just proposed a possible program to give a solid basis to categories at a meeting in Oberwolfach.

It might not seem so but Grothendieck's biography is much more similar to Simone Weil's than to the life of her brother André.
It is true. In spite of the fact that Grothendieck had always lived in unbearable material conditions, while Simone Weil came from a wealthy family, many analogies can be drawn between them. Their anarchistic con-

victions and their preoccupation for being close to the poorest is what linked them. Once Grothendieck told me very proudly that his father had been a political prisoner under 17 different regimes. On the other hand, it was a common joke that Trotsky's 4th International had been founded in Simone Weil's apartment.

In the second part of his life, Grothendieck was dragged by a self-destructive behaviour very similar to Simone Weil's, who died at 34 years of age due to badly cured tuberculosis and a refusal to eat. Both had an excessive way of looking at things and lived a totally personal religious experience: in Simone Weil's case it is a blend of Indian religion and Christianity as seen through the Greek prism, and it is close to Buddhism in Grothendieck's.

In search of the cosmic Galois group

In your works on the concept of space you have conjectured the existence of a "cosmic Galois group", an idea which Grothendieck would have probably liked. What is the meaning of this group?

In science you sometimes have to find a word that strikes, such as "catastrophe", "fractal" or "Noncommutative Geometry". They are words which do not express a precise definition but a program worthy of being developed. What allowed me to formulate the notion of "cosmic Galois group" was having kept a very close eye on the advances in mathematical physics throughout all my career. On the one hand, by introducing a new group Connes and Kreimer achieved a completely new reformulation of the problem of regularizing a family of integrals in such a way that the algebraic relations expressing different physical phenomena were preserved. On the other hand, in some works in which I had collaborated, we were interested in series and integrals representing numbers which generalize the powers of π and zeta values. When studying relations between these numbers, one encounters the motivic Galois group, which provides a kind of transcendental Galois theory. Inspired by the analogy between the two constructions, I suggested that they are actually the same group. Why are they equal? When explicit calculations are made in physics, the constants which appear are the same which number theory treats. In the Standard Model there is a table of about twenty constants, on which the history of our universe depends in a very precise manner. My dream is a total fusion of these ideas which would permit an interpretation of the Connes-Kreimer group as a symmetry group of these fundamental constants.

When one looks at your papers, there is an admirable variety of topics and research domains. How do you choose the problems which you will work on? Do you usually think about different questions at the same time?

It is said that Feynman once explained the following: "It is very easy to be a genius: you have ten problems in mind, you think constantly about them. For each thing that you come across, you wonder *Can this help me?* After some time there will be a box full of theorems". My method is my character: I am curious by nature and everything attracts my attention. Since I have acquired a *savoir faire* in very different directions, when I study a problem I always have several techniques in mind. I am also very interested in questions concerning history and the philosophy of mathematics. André Weil taught me to learn from the mathematicians of the past as if they were our contemporaries. For me the big question is always the same: what guarantees that mathematics tells the truth? How does it tell it? Does it always tell it?

My background, characterised by the contrast between the Alsatian common sense of my grandmother and the somewhat delirious imagination of my father, gave me a huge curiosity for people, countries and books. My 13-year-old grandson once told me: "When I am older I want to be like you. I hope to be as strong as you, travel as much as you and have as many friends as you".

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